Teaching Statement

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1 My Teaching Experiences

During my time at the University of Arizona, I have had the opportunity to teach a variety of courses. Many of my most memorable experiences in Arizona, in fact, occured either in the classroom or with students in my office. I cherish every moment at the chalkboard, and thrive off of the eagerness to learn that I see in my students' eyes. From an early age, I had a passion for teaching mathematics (I was the older brother forcing his younger sister to fill out addition/subtraction workbooks over summer vacation), and that fire burns stronger now that I have had the chance to instruct college students.

Whenever I teach, I bring every ounce of enthusiasm I can muster. This makes my job as a teacher exciting, and I believe it makes my students more eager to learn. From my experience as a student, I certainly know that if the professor does not seem genuinely interested in the material, then it is difficult to enjoy the course and learn effectively. Victoria B., a precalculus student from Spring 2013, wrote a letter to the mathematics department head and my teaching supervisor, stating that I "taught math with the bright-eyed enthusiasm of someone who's had just the right amount of coffee. Every day was something new. Every day was interesting. [...] As a student I will say this: hold on to Stephen as long as you can. He is, without doubt, one of the most impressive talents in the Mathematics department." Alyssa H. wrote me a thank-you card, stating "I received an 'E' in Math 120R when I took it last semester. I swore I would never like math again. I even considered dropping my major because I felt math was impossible. Because I had you as a teacher, I love math again and I will continue to pursue my dreams". Many students inquire about my teaching assignment for the upcoming semester, in hopes that they can enroll in my course. If they cannot, some will still request that I tutor them or stop by my office to get help. Not even halfway through the current semester, Alejandro and Jose asked if I would be teaching vector calculus in the Spring. "I guess this is just a one-hit wonder" they replied, when I told them I would not. On several occasions, others asked if they could bring friends to my lectures, review sessions, or office hours because "I explain the concepts well". As an instructor, I strive to give every student the opportunity to love mathematics.

I taught Math 112 (College Algebra) in the Spring and Fall 2012 terms, Math 120R (Pre-Calculus) in the Spring 2013 term, Math 122B (Calculus I) in the Fall 2013 and Summer 2015 terms, Math 125 ("Accelerated" Calculus I) in the Fall 2014 term, and am currently teaching Math 129 (Calculus II). As an instructor for these courses at the University of Arizona, I am responsible for preparing and delivering lectures, writing and grading quizzes and exams, and assigning and grading homework assignments. I have also served as a teaching assistant for several courses, including Math 323 (Formal Mathematical Reasoning and Writing), Math 196L (Pre-Calculus Supplemental Instruction), Math 196V (Vector Calculus Workshop), and Math 511 (Graduate Algebra Core Course). As a teaching assistant in Math 323 and Math 511, my responsibilities included grading homework and holding office hours, and in the case of Math 511, I held weekly problem sessions. In these sessions, we discussed homework and worked through practice problem sets in preparation for the qualifying exams which are (typically) taken after the first year of the graduate program. The Math 196L and 196V courses have an Inquiry-Based Learning (IBL) setup. We provide the students with problem sets, and they lead the discussions on problem solving strategies. As a teaching assistant, I monitored these discussions, giving hints as needed and asking thought provoking questions. Though I did not have sole leadership in these courses, the teaching assistantship was valuable nonetheless. I experienced the benefits and potential flaws of IBL, a method that I plan to incorporate into my teaching style. I also gained insight on the difficulties that students have in upper division mathematics courses and proof-writing, which will inevitably prepare me for when I handle the reins as the full-time instructor in the future.

2 Technology

I incorporate technology into my lectures on a regular basis. Whether it be calculators, computer programs, or online resources, technology, when implemented properly, can be a tremendously effective teaching tool. In college algebra, pre-calculus, and every calculus course we use graphing calculators with various goals. For example, in college algebra, we use the graphing capabilities to examine the graphs of rational functions, allowing students to make conjectures about the features of the graph (vertical asymptotes, horizontal asymptotes, etc.) as they relate to the equation. In calculus, we use the computational speed and power of the graphing calculators to compute limits as an introduction to the derivative of a function at a point. In the multivariable calculus workshop that I assist, we use tablets and graphing applications such as *Quick Graph Plus* to plot three-dimensional surfaces, tangent planes, and level sets.

At the University of Arizona, we utilize online resources for homework. In my college algebra class we used MyMathLab, and in precalculus and calculus I and II, we used WebAssign. Students have multiple attempts at solutions, and thus, the online homework aids in problem solving as well as clarifying notation. For instance, calculus II students often forget to write "dx" when computing integrals. WebAssign will not give full credit in this situation, but will allow the student to fix the mistake. As a result, I noticed a significant increase in the quality of the submitted written work, which is strategically assigned to be due after the online homework.

In my research, I often use the computer algebra system GAP - Groups, Algorithms, Programming. GAP is a large database containing algebraic objects and a language that permits the manipulation of these objects under an assortment of functions. In the future, I hope to teach a linear algebra or a group theory course, so that I may teach the basics of GAP.

3 Teaching Philosophy

I believe that one of the most important components of my role as a mathematics instructor is connecting with the audience. As a mathematician, I would be exhilarated if all of my students were as excited about studying mathematics as I am. However, this is generally not the case. I believe that it is my job to change these students' opinions, and coax them into appreciating the beauty of mathematics. In lower division courses such as college algebra, I will often collect a survey from students at the beginning of the semester. On this survey, I obtain general background information (name, hobbies, major, etc.), as well as their past experiences in mathematics, both positive and negative. I then compare the responses to the end of term course evaluations. In many cases, I am pleased to observe a significant turnaround in their attitude toward mathematics. "He did a much better job than my teacher last semester for 112. I learned a good amount of info in this class." I feel a sense of great accomplishment when a student who had a poor experience in the past with mathematics tells me that they learned valuable information in my class.

Another ingredient in connecting with my audience is knowing them on a more personal level, and recognizing their idiosyncrasies. Every semester, I memorize my students' names before the term begins, in an effort to make them feel comfortable in the classroom from the get-go. In the aforementioned start of the term surveys, I obtain information regarding every student's major, and this allows me to use real-life examples to explain abstract concepts. For example, in college algebra we have a lot of business majors, and hence I will use profit, revenue, and cost to explain possible operations on functions, rather than simply stating the definition of the sum and difference of functions $f+g$ or $f-g$. Students can appreciate the curriculum, and the class no longer becomes a game of "passing the last math class I'll ever have to take". Instead, many students realize that mathematics is integrated into our daily lives, even if it is not so obvious. One student remarked "The more math I practice, the better I get. I could not have done it without Stephen. His ability to teach has aided me in chemistry, physics and elsewhere. He did not teach me to do - he taught me to think. He believed I was smart enough to learn, even when I was not sure myself." One of my goals as an instructor is to alleviate any tension that students feel toward mathematics, and part of this is recognizing typical errors and idiosyncracies. Many college algebra students will mistakenly simplify $\sqrt{x^2+y^2}$ to $x+y$ or

$\frac{x+y}{x}$ to y. Whether it is in a one-on-one situation in office hours or in front of the classroom, I make it a point to gently remind everyone that these simplifications are incorrect, and give counterexamples to disprove the statements. Lauren G. was one of my college algebra students that gained a lot of confidence as the semester progressed. She did poorly on the first exam as a result of these common errors, but attended office hours nearly every day. By the end of the semester, she no longer made these errors, and scored the highest grade in my section on the (department constructed) final exam, finishing with an A in the course. Nevertheless, I understand that we will not always solve our problems on the first try. Thus, when we are in class and I ask for suggestions on strategies for solving problems, I will try to finish the problems using the students' ideas, even if I know the work will be fruitless. For example, in calculus II when we integrate products of functions using integration by parts, I will let students choose whatever they want for u and dv, and we will attempt to integrate. At first, their choices generally do not yield the desired consequence, but I use this to explain strategies for appropriately choosing the best u and dv. Though this is frustrating to some students, it is an important facet of the learning process, and encourages them to never give up on a problem simply because their first attempt failed.

Approachability is critical. I never want to give the impression that the material that I teach is trivial, to ensure that students feel comfortable asking questions when they are confused. To quote Victoria again, "This is an individual who can finish a pre-calculus exam in eight minutes - yet, he never taught as if the subject matter were beneath him. He never ridiculed any question that was asked of him, and many times used those questions as a means to alternatively explain a problem.[...]Without knowing that a girl raised to do nothing had taken such a massive gamble, he always gave me more than he ever had to. He treated me with extraordinary respect. He was patient and attentive and never derided me for what little I knew or when I made mistakes (which was often). I cannot begin to express how grateful I am." I demand that the classroom is a safe place, where everyone is respected. As an undergraduate, I had instructors who would belittle students when they asked questions, and as a result, I was too uncomfortable to attend office hours. This tends to create a hostile environment, where learning cannot take place at its full potential. I take pride in the fact that nearly every students responds "strongly agree" when asked if they were treated with respect on their course evaluations. One student suggested that I "keep being friendly to your students" - I certainly plan to continue showing respect toward every one of my students, as this is crucial if I want my to students to respect me and, more importantly, the mathematics that I love to share.

Expanding on approachability, I believe that availability is necessary. I never hesitate to put my students' needs before mine. If it means coming to my office early, staying late, or skipping a seminar to talk with a student who cannot meet at another time, I will do everything in my power to assist my students. John C. was unable to attend my scheduled office hours, but after he broke his arm in a skiing accident, he asked if I would be willing to meet with him at a different time to catch up on the work he missed. I met with John at 8AM, three days a week, for nearly a third of the semester to help him earn the B that he wanted in the class. "He once spent two hours outside of his regularly scheduled office hours to explain some features of trigonometry with which I was struggling. He hosted a final exam review for several hours during a weekend, and even discussed my final exam errors with me. These are things he never had to do, and yet he did so, without hesitation. If it was any imposition to him at all (it would seem a Ph.D. candidate's time is at a premium) he never made that apparent." When asked "What did you especially like about this course?" on the end of semester evaluations, many students comment on my willingness to help. "Stephen was always available to help when needed." "I liked that Stephen was always going out of his way to help his students and held extra review sessions before each test to make sure we did well."

In the classroom, I encourage both individual thinking and group collaboration. At the beginning of every class period, I will write a few questions on the whiteboard covering material discussed in the previous lecture. For the first few minutes, I treat this almost as though it is a quiz, and students attempt the problems on their own. Then, I allow them to talk things over with the people sitting near them. There are several advantages to this process. Students are focused from the very beginning of class, and get a chance to look through their notes from the last class. When they move on to the group discussion, students can verbalize their thought processes with their peers, solidifying their understanding and clarifying confusion. Regardless

of the correctness of the solution, the dialogue boosts the confidence of all members involved. Many times, I will ask volunteers to present their solutions to the class on the whiteboard, and talk through their train of thought. I have found that teaching mathematics is the best way to truly understand mathematics, and this process allows my students to learn through teaching. On a related note, in the vector calculus workshop that I monitor, we structure the discussion so that students are in groups of four. One person writes on the board while the others provide constructive criticism, and the roles switch regularly. I hope to implement this arrangement more often in the future, as it promotes lively, yet positive debate. Furthermore, it ensures that everyone is involved in the learning process. I believe that students gain more knowledge if there is a discussion, rather than just a lecture.

My teaching experiences have helped to shape who I am as an instructor, as a researcher, and as a mathematician. My primary goal is to discover new mathematics and educate others at all levels so that they may enjoy the beautiful, yet difficult subject. The opportunities presented to me at the University of Arizona have influenced my philosophy toward teaching, and I have realized that there are essential requirements when creating an effective learning environment. My passion and enthusiasm for teaching and my respect for students and peers alike provide for a culture where anyone can succeed. As Victoria put it, bringing tears to my eyes, "if you opt to take the road less traveled by, it's nice to know there's someone with a vested interest in helping you while you're there. Frost was only half right. Perhaps it's not simply taking that road - perhaps it's the luck of crossing paths with somebody remarkable that makes the difference. For me, Stephen Trefethen is that person."